Kepler's Laws and Planetary Motion

Read from Lesson 4 of the Circular and Satellite Motion chapter at The Physics Classroom:

http://www.physicsclassroom.com/Class/circles/u6l4a.cfm

MOP Connection: Circular Motion and Gravitation: sublevel 10

1. Kepler's first law of planetary motion states that _____. Choose one. Answer: B
   a. the Sun is at the center of the solar system
   b. planets orbit the Sun in elliptical orbits, with the Sun located at one focus
   c. planets orbit the Sun in circular orbits, with the Sun located at the center
   d. gravity provides the force that holds the planets in orbit about the Sun

2. Kepler's second law of planetary motion states that a line connecting a planet to the Sun _____. Choose one. Answer: C
   a. is longest in winter and shortest in summer
   b. sweeps out more area during a winter month than during the summer month
   c. sweeps out the same amount of area in any two equal periods of time
   d. sweeps out the same amount of area regardless of the planet.

3. A planet would move ____. Answer: B
   a. at the same speed at all times during its orbit about the Sun
   b. at faster speeds when positioned closer to the Sun during its orbit
   c. at slower speeds when positioned closer to the Sun during its orbit

4. Kepler's third law of planetary motion states that the ratio of ____. Answer: D
   a. the orbital period to the orbital radius is the same for all planets
   b. the orbital periods of any two planets equals the ratio of the orbital radii
   c. all planets would orbit with the same orbital period
   d. the period squared to the radius cubed is the same ratio for all planets

5. A planet that is further from the Sun would take ___more___ time to orbit the Sun compared to planets that are closer to the Sun.
   a. more  b. less  c. the same amount of

6. Planetary data for the nine planets are shown below. Radius and period data are expressed relative to the Earth's radius and period.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Period (Earth years)</th>
<th>Ave. Radius (astron. units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.241</td>
<td>0.39</td>
</tr>
<tr>
<td>Venus</td>
<td>0.615</td>
<td>0.72</td>
</tr>
<tr>
<td>Earth</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Mars</td>
<td>1.88</td>
<td>1.52</td>
</tr>
<tr>
<td>Jupiter</td>
<td>11.8</td>
<td>5.20</td>
</tr>
<tr>
<td>Saturn</td>
<td>29.5</td>
<td>9.54</td>
</tr>
<tr>
<td>Uranus</td>
<td>84.0</td>
<td>19.18</td>
</tr>
<tr>
<td>Neptune</td>
<td>165</td>
<td>30.06</td>
</tr>
<tr>
<td>Pluto</td>
<td>248</td>
<td>39.44</td>
</tr>
</tbody>
</table>
Taking two planets at a time, compare the ratio of the square of the period to the ratio of the cube of their radius.

There are no units for the ratios below. All answers are reported to the third significant digit.

\[
\begin{align*}
\text{(TMars / TEarth)}^2 &= 3.53 \\
\text{(TJupiter / TEarth)}^2 &= 139 \\
\text{(TNeptune / TUranus)}^2 &= 3.86 \\
\text{(TPluto / TUranus)}^2 &= 8.72 \\
\text{(TUranus / TEarth)}^2 &= 7060 \\
\text{(TEarth / TVenus)}^2 &= 2.64 \\
\text{(TNeptune / TMars)}^2 &= 7.70 \times 10^3
\end{align*}
\]

(RMars / REarth)^3 = 3.51
(RJupiter / REarth)^3 = 141
(RNeptune / RUranus)^3 = 3.85
(RPluto / RUranus)^3 = 8.69
(RUranus / REarth)^3 = 7060
(REarth / RVenus)^3 = 2.68
(RNeptune / RMars)^3 = 7.73 \times 10^3

Answers will vary. One should notice though that the \(T^2\) ratio is approximately equal to the \(R^3\) ratio.

7. Complete the following statements.

The questions below are challenging, yet strike at the heart of the meaning of Kepler's third law. For any two planets orbiting the same central body (e.g., the Sun), the square of the ratio of their periods is equal to the cube of the ratio of their radii of orbit.

a. If planet A is twice as far from the Sun as planet B, then the period of its orbit will be __2.83___ times as long. The square root of the radius ratio cubed (2.0^3).

b. If planet A is three times as far from the Sun as planet C, then the period of its orbit will be __5.20___ times as long. The square root of the radius ratio cubed (3.0^3).

c. If planet A is four times as far from the Sun as planet C, then the period of its orbit will be __8.00___ times as long. The square root of the radius ratio cubed (4.0^3).

d. If planet A is five times as far from the Sun as planet C, then the period of its orbit will be __11.2___ times as long. The square root of the radius ratio cubed (5.0^3).

8. If a small planet were located eight times as far from the sun as the Earth’s distance from the sun, how many years would it take the planet to orbit the sun. \textbf{PSYW}

The cube of the ratio of the planets' radii is equal to the square of the ratio of the planets' periods.

\[
\left(\frac{T_{\text{planet}}}{T_{\text{Earth}}}\right)^2 = \left(\frac{R_{\text{planet}}}{R_{\text{Earth}}}\right)^3 = (8.0)^3 = 512
\]

And so the ratio of the periods is the square root of 512 or 22.6. Since the Earth’s period is 1 year, the period of this small planet is 22.6 times larger. We refer to this as \textbf{22.6 earth-years}.  

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